

Available online at www.sciencedirect.com



Journal of Network and Computer Applications 30 (2007) 60-80



www.elsevier.com/locate/jnca

# Adaptive anomaly detection with evolving connectionist systems

Yihua Liao<sup>a,\*</sup>, V. Rao Vemuri<sup>a,b</sup>, Alejandro Pasos<sup>b</sup>

<sup>a</sup>Department of Computer Science, University of California at Davis, One Shields Avenue, Davis, CA 95616, USA <sup>b</sup>Department of Applied Science, University of California at Davis, One Shields Avenue,

Department of Applica Science, entering of California at Davis, one Snielas A Davis, CA 95616, USA

Received 28 June 2005; accepted 28 June 2005

#### Abstract

Anomaly detection holds great potential for detecting previously unknown attacks. In order to be effective in a practical environment, anomaly detection systems have to be capable of online learning and handling concept drift. In this paper, a new adaptive anomaly detection framework, based on the use of unsupervised evolving connectionist systems, is proposed to address these issues. It is designed to adapt to normal behavior changes while still recognizing anomalies. The evolving connectionist systems learn a subject's behavior in an online, adaptive fashion through efficient local element tuning. Experiments with the KDD Cup 1999 network data and the Windows NT user profiling data show that our adaptive anomaly detection systems, based on Fuzzy Adaptive Resonance Theory (ART) and Evolving Fuzzy Neural Networks (EFuNN), can significantly reduce the false alarm rate while the attack detection rate remains high.

© 2005 Elsevier Ltd. All rights reserved.

*Keywords:* Adaptive anomaly detection; Online learning; Concept drift; Evolving connectionist systems; Fuzzy ART; EFuNN

1084-8045/ $\ensuremath{\$}$  - see front matter  $\ensuremath{\textcircled{0}}$  2005 Elsevier Ltd. All rights reserved. doi:10.1016/j.jnca.2005.08.005

<sup>\*</sup>Corresponding author.

*E-mail addresses:* yhliao@ucdavis.edu (Y. Liao), rvemuri@ucdavis.edu (V.R. Vemuri), apasos@ucdavis.edu (A. Pasos).

# 1. Introduction

Computer security vulnerabilities and flaws are being discovered every day. Given the rapid increase in connectivity and accessibility of computer systems in today's society, computer intrusions and security breaches are posing serious threats to national security as well as enterprise interests. As one of the two general approaches to intrusion detection, anomaly detection has been under intensive study for the last two decades (McHugh, 2001). Unlike the alternative approach, misuse detection, which generates an alarm when a known attack signature is matched, anomaly detection analyzes a set of characteristics of the monitored system (or users) and identifies activities that deviate from the normal behavior. It is assumed that such deviations may indicate that an intrusion or attack exploiting vulnerabilities has occurred (or may still be occurring). Any observable behavior of the system can be used to build a model of the normal operation of the system. Audit logs, network traffic, user commands and system calls are all common choices. While misuse detection is effective in recognizing previously known intrusion types, anomaly detection holds great potential for detecting attempts to exploit new and unforeseen vulnerabilities, as well as "abuse of privileges" types of attacks by legitimate users, the so-called "insider threat."

The goal of anomaly detection is to identify anomalous activities (i.e., rare, unusual events) in the audit data stream accurately and in a timely fashion. Over the years, many machine learning and statistical methods have been proposed for anomaly detection, including rule-based approaches (Lee and Stolfo, 2000), immunological-based approaches (Forrest et al., 1996), neural nets (Debar et al., 1992; Ghosh and Schwartzbard, 1999), instance-based approaches (Lane and Brodley, 1999; Liao and Vemuri, 2002), clustering methods (Eskin et al., 2002; Lazarevic et al., 2003), probabilistic learning methods (Eskin, 2000; Mahoney and Chan, 2002), multi-covariance analysis (Javitz and Valdes, 1994), and so on. However, in real-world applications, anomaly-based intrusion detection systems (IDSes) tend to give less than satisfactory performance and generate excessive false alarms. Summarized below are three main reasons.

First, there is a fundamental asymmetry in anomaly detection problems: normal activity is common and intrusive activity is rare. One often faces a training set consisting of a handful of attack examples and much more normal examples, or no attack example at all. The skewed class distribution presents difficult challenges to machine learning methods, especially to discriminative methods that try to learn the distinction between normal and abnormal classes.

Second, many previous anomaly detection approaches involve off-line learning, where data is collected, manually labeled, and then provided to a learning algorithm to build the model of the normal (and abnormal) behavior. In a practical environment, an IDS is operating continuously and new data is available at every time instant. Thus, it may be prohibitively expensive to frequently update the training corpus with clean labeled new batch of audit data and re-train the IDS.

Last, most machine learning and statistical methods assume that the training examples are drawn from a stationary distribution. In practice, however, system and network activities as well as user behavior could change for bonafide reasons. Therefore, the normal behavior may not be strictly predictable in the long term. This problem, known as *concept drift* in machine learning literature, presents a significant challenge for anomaly detection. An effective anomaly detection system should be capable of adapting to normal behavior changes while still recognizing anomalous activities. Otherwise, large amount of false alarms would be generated if the normal behavior model of the IDS failed to change accordingly to accommodate the new patterns.

In this paper, we present an adaptive anomaly detection framework that are suitable for dynamic, changing environments. Our framework employs unsupervised evolving connectionist systems to learn system, network or user behavior in an online, adaptive fashion without a priori knowledge of the underlying non-stationary data distributions. Normal behavior changes are efficiently accommodated while anomalous activities can still be identified.

Adaptive learning and evolving connectionist systems are an active area of artificial intelligence research. Evolving connectionist systems are artificial neural networks that resemble the human cognitive information processing models. They are stable enough to retain patterns learned from previously observed data while being flexible enough to learn new patterns from new incoming data. Due to their self-organizing and adaptive nature, they provide powerful tools for modeling evolving processes and knowledge discovery (Kasabov, 2002).

Our adaptive anomaly detection framework performs one-pass clustering of the input data stream that represents a monitored subject's behavior patterns. Each new incoming instance is assigned to one of the three states: *normal, uncertain* and *anomalous*. Two different alarm levels are defined to reduce the risk of false alarming. We evaluated our adaptive anomaly detection systems, based on the Fuzzy Adaptive Resonance Theory (Fuzzy ART) (Carpenter et al., 1991) and Evolving Fuzzy Neural Networks (EFuNN) (Kasabov, 2001), over two types of data sets, the KDD Cup 1999 network data (The third international knowledge discovery and data mining tools competition (KDD cup 1999) data, 1999) and Windows NT user profiling data. Our experiments show that both evolving connectionist systems are able to adapt to user or network normal behavior changes and at the same time detect anomalous activities. Compared to support vector machines (SVM)-based static learning, our adaptive anomaly detection systems significantly reduced the false alarm rate.

The rest of this paper is organized as follows: in Section 2, we review some related work on adaptive anomaly detection. Section 3 presents our adaptive framework and a brief introduction to Fuzzy ART and EFuNN. Section 4 details our experiments with the KDD Cup 1999 network data and the Windows NT user profiling data. Section 5 contains further discussions. Finally, we summarize our conclusions and future work in Section 6.

# 2. Related work

62

To handle concept drift and non-stationary data distributions, a common practice is to forget out-of-data statistics of the data and favor recent events using a decay or aging factor. For example, NIDES (Javitz and Valdes, 1994) compares a user's short-term

behavior to the user's long-term behavior. The user profiles keep statistics such as frequency table, means and covariance, which are constantly aged by multiplying them by an exponential decay factors. This method of aging creates a moving time window for the profile data, so that the new behavior is only compared to the most recently observed behaviors that fall into the time window. Similarly, SmartSifter (Yamanishi et al., 2004) employs discounting algorithms to gradually fade the effect of past examples. Mahoney and Chan (2002) took training decay to the extreme by discarding all events before the most recent occurrence. There is one theoretical and one practical problem with this aging or time window approach. Theoretically, no justification has been provided for the assumption that a user's behavior changes gradually. Notwithstanding this theoretical gap, the decay factor is usually chosen in an ad hoc manner. By contrast, our evolving connectionist systems are able to adapt to normal behavior changes without losing earlier information.

There were a few other previous efforts on adaptive intrusion detection. Teng et al. (1990) proposed a time-based inductive learning approach to perform adaptive real-time anomaly detection. Sequential rules were generated dynamically to adapt to changes in a user's behavior. Lane and Brodley (1998) proposed a nearest-neighbor classifiers-based online learning scheme and examined the issues of incremental updating of system parameters and instance selection. Finite mixture models were employed in Eskin et al. (2000) to generate adaptive probabilistic models and detect anomalies within a data set. Fan (2001) used ensembles of classification models to adapt existing models in order to detect newly established patterns. Hossain and Bridges (2001) proposed a fuzzy association rule mining architecture for adaptive anomaly detection.

Compared to previous statistical and rule-learning-based adaptive anomaly detection systems, our framework does not require a priori knowledge of the underlying data distributions. Through the use of evolving connectionist systems, it provides efficient adaptation to new patterns in a dynamic environment. Unlike other neural networks that have been applied to intrusion detection (e.g., Debar et al. (1992); Ghosh and Schwartzbard (1999)) as "black boxes," our evolving connectionist systems can provide knowledge (i.e., the weight vectors) to "explain" the learned normal behavior patterns.

Our approach also falls into the category of unsupervised anomaly detection (Eskin et al., 2002; Lazarevic et al., 2003) as it does not require the knowledge of data labels. However, our algorithms assign each instance into a cluster in an online, adaptive mode. No distinction between training and testing has to be made. Therefore, the period of system initialization during which all behaviors are assumed normal is not necessary.

Another research project closely related to ours is ADMIT (Sequeira and Zaki, 2002), which uses semi-incremental clustering techniques to create user profiles. Different types of alarms are also introduced.

## 3. Adaptive anomaly detection framework

In addressing the problem of adaptive anomaly detection two fundamental questions arise: (a) How to generate a model or profile that can concisely describe a

subject's normal behavior, and more importantly, can it be updated efficiently to accommodate new behavior patterns? (b) How to select instances to update the model without introducing noise and incorporating abnormal patterns as normal? Our adaptive anomaly detection framework addresses these issues through the use of online unsupervised learning methods, under the assumption that normal instances cluster together in the input space, whereas the anomalous activities correspond to outliers that lie in sparse regions of the input space. Our framework is general in that the underlying clustering method can be any online unsupervised evolving connectionist system and it can be used for different types of audit data. Without loss of generality, we assume the audit data that is continuously fed into the adaptive anomaly detection system has been transformed into a stream of input vectors after pre-processing, where the input features describe the monitored subject's behavior.

The evolving connectionist systems are designed for modeling evolving processes. They operate continuously in time and adapt their structure and functionality through a continuous interaction with the environment (Kasabov, 2002). They can learn in unsupervised, supervised or reinforcement learning modes. The online unsupervised evolving connectionist systems provide one-pass clustering of an input data stream, where there is no predefined number of different clusters that the data belong to.

A simplified diagram of an evolving connectionist system for online unsupervised learning is given in Fig. 1(a) (some systems such as EFuNN may have an additional fuzzy input layer, shown in Fig. 1(b), which represents the fuzzy quantization of the original inputs with the use of membership functions (Jang et al., 1997). A typical unsupervised evolving connectionist system consists of two layers of nodes: an *input* layer that reads the input vectors into the system continuously, and a *pattern* layer (or cluster layer) representing previously learned patterns. Each pattern node corresponds to a cluster in the input space. Each cluster, in turn, is represented by a weight vector. Then the subject's normal behavior profile is conveniently described as a set of weight vectors that represent the clustering of the previous audit data.

A distance measure has to be defined to measure the mismatch between a new instance (i.e., a new input vector) and existing patterns. Based on the distance measure, the system either assigns an input vector to one of the existing patterns and updates the pattern weight vector to accommodate the new input, or otherwise creates a new pattern node for the input. The details of clustering vary with different evolving connectionist systems.

In order to reduce the risk of false alarms (classifying normal instances as abnormal), we define three states of behavior patterns (i.e., the pattern nodes of the evolving connectionist system): *normal, uncertain* and *anomalous*. Accordingly, each instance is labeled as either *normal, uncertain* or *anomalous*. In addition, the alarm is differentiated into two levels: *Level* 1 *alarm* and *Level* 2 *alarm*, representing different degrees of anomaly. As illustrated in Fig. 2, a new instance is assigned to one of the existing *normal* patterns and labeled *normal* if the similarity between the input vector and the *normal* pattern is above a threshold (the *vigilance* parameter). Otherwise, it is *uncertain*. The *uncertain* instance is either assigned to one of the existing *uncertain* patterns if it is close enough to that *uncertain* pattern, or becomes the only member

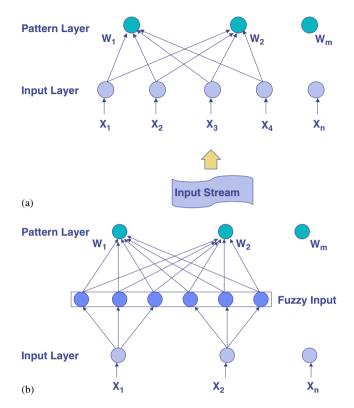


Fig. 1. (a) A simplified diagram of an evolving connectionist system for unsupervised learning. The system has n input nodes and m pattern nodes. There is a connection from each input node to every pattern node. Some connections are not shown in the figure. (b) An evolving connectionist system that has an additional fuzzy input layer. The task of the fuzzy input nodes is to transfer the input values into membership degrees.

of a new *uncertain* pattern. A *Level* 1 *alarm* is triggered whenever a new *uncertain* pattern is created as the new instance is different from all the learned patterns and thus deserves special attention. At this point, some preliminary security measures need to be taken. However, one cannot draw a final conclusion yet. The new instance can be truly anomalous or merely the beginning of a new normal behavior pattern, which will be determined by the subsequent instances. After the processing of a certain number (the  $N_{watch}$  parameter) of the subsequent instances in the same manner, if the number of members of an *uncertain* pattern reaches a threshold value (the *Min<sub>count</sub>* parameter), the *uncertain* pattern becomes a *normal* pattern and the labels of all its members are changed from *uncertain* to *normal*. This indicates that a new behavior pattern has been developed and incorporated into the subject's normal behavior profile as enough instances have shown the same pattern. On the other hand, after  $N_{watch}$  subsequent instances, any *uncertain* pattern with less than *Min<sub>count</sub>* members will be destroyed and all its members are labeled *anomalous*. This will make

- (1) Assign the first instance to a new uncertain pattern and label it *uncertain*.
- (2) Consider the next instance.

66

- (a) If the similarity between this instance and one of the existing *normal* patterns is above *vigilance*, assign it to the *normal* pattern and label it *normal*.
- (b) If the instance is close enough to one of the existing *uncertain* patterns, assign it to the *uncertain* pattern and label it *uncertain*.
- (c) Otherwise, the instance becomes the only member of a new *uncertain* pattern. A *Level 1 alarm* is triggered.
- (d) Increase the age of each existing *uncertain* pattern by 1.
- (e) for each *uncertain* pattern whose age has reached  $N_{watch}$ , if it has more than  $Min_{count}$  members, it becomes a *normal* pattern and change the labels of all its members from *uncertain* to *normal*. Otherwise, the *uncertain* pattern is destroyed and all its members are labeled *anomalous*. Level 2 alarms are issued.
- (3) Repeat step 2 to process subsequent instances.

Fig. 2. Pseudo-code for adaptive anomaly detection.

sure that anomalous patterns, corresponding to the sparse regions in the input space, will not be included into the normal profile. A *Level 2 alarm* is issued when an instance is labeled *anomalous* and further response actions are expected.

The main tunable parameters of an adaptive anomaly detection system are summarized as follows:

- *Vigilance*  $\rho$ : This threshold controls the degree of mismatch between new instances and existing patterns that the system can tolerate.
- Learning rate  $\beta$ : It determines how fast the system should adapt to a new instance when it is assigned to a pattern.
- $N_{watch}$ : It is the period that the system will wait before making a decision on a newly created *uncertain* pattern.
- *Min<sub>count</sub>*: The minimum number of members that an *uncertain* pattern should have in order to be recognized as a *normal* pattern.

Our framework does not require a priori knowledge of the number of input features. When a new input feature is presented, the system simply adds a new input node to the input layer and connections from this newly created input node to the existing pattern nodes. This can be very important when the features that describe a subject's behavior grow over time and cannot be foreseen in a dynamic environment. Similarly, accommodation of a new pattern is efficiently realized by creating a new pattern node and adding connections from input nodes to this new pattern node. The rest of the structure remains the same.

With the framework, the learned normal profile is expressed as a set of weight vectors representing the coordinates of the cluster centers in the input space. These

weight vectors can be interpreted as a knowledge presentation that can be used to describe the subject's behavior patterns, and thus they can facilitate understanding of the subject's behavior. The weight vectors are stored in the long-term memory of the connectionist systems. Since new instances are compared to all previously learned patterns, recurring activities would be recognized easily.

While the underlying clustering method of the adaptive anomaly detection framework can be any unsupervised evolving connectionist system, Fuzzy ART and the unsupervised learning version of EFuNN are adapted for anomaly detection in this paper. Both of them are conceptually simple and computationally fast. Furthermore, they cope well with fuzzy data, and the fuzzy distance measures help to smooth the abrupt separation of normality and abnormality of a subject's behavior. Below is a brief introduction to Fuzzy ART and EFuNN.

#### 3.1. Fuzzy ART

Fuzzy ART (Carpenter et al., 1991) is a member of the ART neural network family (Carpenter and Grossberg, 1991). It incorporates computations from fuzzy set theory (Jang et al., 1997) into the ART 1 neural network. It is capable of fast stable unsupervised category learning and pattern recognition in response to arbitrary input sequences.

Fuzzy ART clusters input vectors into patterns based on two separate distance criteria, *match* and *choice*. For input vector X and pattern j, the *match* function is defined by

$$S_j(X) = \frac{|X \wedge W_j|}{|X|},\tag{1}$$

where  $W_j$  is the weight vector associated with pattern *j*. Here, the fuzzy AND operator  $\wedge$  is defined by

$$(X \wedge Y)_i \equiv \min(x_i, y_i) \tag{2}$$

and the norm  $|\cdot|$  is defined by

$$|X| \equiv \sum_{i} |x_{i}|. \tag{3}$$

The choice function is defined by

$$T_j(X) = \frac{|X \wedge W_j|}{\alpha + |W_j|},\tag{4}$$

where  $\alpha$  is a small constant.

For each input vector X, Fuzzy ART assigns it to the pattern j that maximizes  $T_j(X)$  while satisfying  $S_j(X) \ge \rho$ , where  $\rho$  is the vigilance parameter,  $0 \le \rho \le 1$ . The weight vector  $W_j$  is then updated according to

$$W_{j}^{(new)} = \beta(X \wedge W_{j}^{(old)}) + (1 - \beta)W_{j}^{(old)},$$
(5)

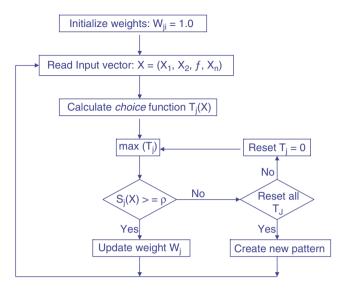


Fig. 3. Flow graph representation of the Fuzzy ART algorithm.

where  $\beta$  is the *learning rate* parameter,  $0 \le \beta \le 1$ . If no such pattern can be found, a new pattern node is created. This procedure is illustrated in Fig. 3.

In order to avoid the pattern proliferation problem, Fuzzy ART uses a *complement coding* technique to normalize the inputs. The complement of vector X, denoted by  $X^c$ , is defined by

$$(X^c)_i \equiv 1 - x_i. \tag{6}$$

For an *n*-dimensional original input X, the complemented coded input X' to the Fuzzy ART system is the 2*n*-dimensional vector

$$X' = (X, X^c) \equiv (x_1, x_2, \dots, x_n, x_1^c, x_2^c, \dots, x_n^c).$$
<sup>(7)</sup>

# 3.2. EFuNN

EFuNN is one of the evolving connectionist systems developed by Kasabov (2001) that is capable of modeling evolving processes through incremental, online learning. It has been successfully applied to bio-informatics, speech and image recognition (Kasabov, 2002). The original EFuNN has a five-layer structure. Here, we only use its first three layers for unsupervised learning (Fig. 1(b)).

The fuzzy input layer transfers the original input values into membership degrees with a membership function attached to the fuzzy input nodes. The membership function can be triangular, Gaussian, and so on. The number and the type of the membership function can be dynamically modified during the evolving process. In this research we used the triangular membership function. Unlike Fuzzy ART, EFuNN groups input vectors into patterns based on one distance measure only, the local normalized fuzzy distance between a fuzzy input vector  $X_f$  and a weight vector  $W_i$  associated with pattern j, which is defined by

$$D(X_f, W_j) = \frac{|X_f - W_j|}{|X_f + W_j|},$$
(8)

where  $|\cdot|$  denotes the same vector norm defined in Fuzzy ART. The local normalized fuzzy distance between any two fuzzy membership vectors is within the range of [0, 1].

The rest of the clustering algorithm of EFuNN is very similar to that of Fuzzy ART. When a new input vector X is presented, EFuNN calculates the corresponding fuzzy input vector  $X_f$  and evaluates the normalized fuzzy distance between  $X_f$  and the existing pattern weight vectors. The activation of the pattern node layer A is then calculated. The activation of a single pattern node j is defined by

$$A(j) = f(D(X_f, W_j)), \tag{9}$$

where *f* can be a simple linear function, for example,  $A(j) = 1 - D(X_f, W_j)$ . EFuNN finds the closest pattern node *j* to the fuzzy input vector that has the highest activation value A(j). If  $A(j) \ge \rho$ , where  $\rho$  is the *vigilance* parameter (the original EFuNN paper named it *sensitivity threshold* (Kasabov, 2001)), the new input is assigned to the *j*th pattern and the weight vector  $W_j$  is updated according to the following vector operation:

$$W_{j}^{(new)} = W_{j}^{(old)} + \beta (X_{f} - W_{j}^{(old)}),$$
(10)

where  $\beta$  is the *learning rate*. Otherwise, a new pattern node is created to accommodate the current instance X.

The parameters  $\rho$  and  $\beta$  can be static, or they can be self-adjustable while the structure of EFuNN evolves. They can hold the same values for all the patterns, or they can be pattern-specific so that the pattern node that has more instance members will change less when it accommodates a new instance. In our early implementation, all the pattern nodes share the same static  $\rho$  and  $\beta$  values.

## 4. Experiments

In this section we describe some experiments. The emphasis of the experiments is on the understanding of how Fuzzy ART and EFuNN-based adaptive anomaly detection systems work in practice. One objective of our experiments is to observe the influence of variability of the tunable parameters on the performance of an anomaly detection system. Another objective of the experiments is to compare SVMbased static learning and evolving connectionist system-based adaptive learning.

## 4.1. Static learning via support vector machines

SVM is a relatively new and state-of-the-art classification method pioneered by Vapnik (1998). It is based on the so-called *structural risk minimization* principle, which minimizes an upper bound on the generalization error. The method performs a mapping from the input space to a higher-dimensional feature space through the use of a kernel function. It separates the data in the feature space by means of a maximum margin hyperplane. Schölkopf et al. (2001) proposed a method of adapting the SVM paradigm to the one-class problem. The origin of the coordinate system, after transforming the feature via a kernel, is treated as the only member of the second class. Training a SVM is equivalent to solving a linearly constrained quadratic programming problem.

In our experiments, we used SVM to demonstrate the weakness of static learning and the importance of adaptive learning. SVM was employed to learn a model (i.e., support vectors) that fits the training data set. The model was then tested on the testing data set without any update (thus it is static learning). SVM is optimal when the data are independent and identically distributed (i.i.d.). If there was concept drift between the training data set and the testing data set, SVM would generate classification errors. Adaptive learning can adapt to concept changes incrementally and learn new patterns when new testing instances are presented to the learning system. Therefore the classification accuracy is improved.

In our research, we used LIBSVM (version 2.35) (Chang and Lin, 2001), an integrated tool for SVM classification and regression.

#### 4.2. Cost function

To facilitate performance comparison among different methods, we used the cost function

$$Cost = (1 - hit \ rate) + \gamma * false \ positive \ rate, \tag{11}$$

where the hit rate is the rate of detected intrusions (attacks or masquerades), the false positive rate is the probability that a normal instance is classified as *anomalous*, and the parameter  $\gamma$  represents the relative cost difference between a false alarm and a miss. There is no obvious way to determine the value of  $\gamma$ , since the cost of a false alarm as well as the cost of a miss will vary from one environment to another. Here we set the  $\gamma$  value to 6, which was used in Maxion and Townsend (2002), while other values are certainly applicable. Varying the tunable parameters' values results in different hit rates and false positive rates, and, subsequently, different cost values. The lower cost, the better performance an intrusion detection system has.

## 4.3. Network intrusion detection

We conducted a series of experiments on a subset of the data set KDD Cup 1999 (The third international knowledge discovery and data mining tools competition (KDD cup 1999) data, 1999) prepared for network intrusion detection. Many methods have been tested with this popular data set for supervised intrusion detection. The data labels were usually used for training the learning systems. Our evolving connectionist systems, however, do not rely on the data labels. They build network connection patterns incrementally in an online unsupervised learning mode. Therefore, they are not directly comparable to previously proposed supervised learning methods.

The 1999 KDD Cup network traffic data are connection-based. Each data record, described by 7 symbolic attributes and 34 continuous attributes, corresponds to a TCP/IP connection between two IP addresses. In addition, a label is provided indicating whether the record is normal or it belongs one of the four attack types (*Probe, DoS, U2R* and *R2L*). The symbolic attributes that have two possible values (e.g., *logged\_in*) were represented by a binary entry with the value of 0 or 1. For symbolic attributes that have more than two possible categorical values, we used multiple entries to encode them in the vector representation, one entry for each possible value. The entry corresponding to the category value has a value of 1 while the other entries are set to 0. The attribute *service* has 41 types, and we further classified them into {*http, smtp, ftp, ftp\_data, others*} to reduce the vector dimensions. The resulting feature vectors have a total of 57 dimensions.

Since different continuous attributes were measured on very different scales, the effect of some attributes might be completely dwarfed by others that have larger scales. Therefore, we scaled the attributes to the range of [0, 1] by calculating

$$X_i = \frac{v_i - \min(v_i)}{\max(v_i) - \min(v_i)},\tag{12}$$

where  $v_i$  is the actual value of attribute *i*, and the maximum and minimum are taken over the whole data set. However, we are aware that this scaling technique would not work if the maximum and minimum values are not known a priori.

We formed a subset of the original data set consisting of 97277 normal connections and 9199 attacks by randomly sampling. We then conducted two experiments with this subset. The first experiment (Exp. 1) was designed to test our evolving connectionist systems. In the data stream of Exp. 1, the attack examples randomly drawn from the 9199 attacks were inserted into the 97277 normal examples with a 1% probability. Fuzzy ART and EFuNN were employed to model the network connections on the fly from an empty set of normal patterns and detect the intrusions in the data stream. For the second experiment (Exp. 2), the training data set and testing data set were formed to compare the performance between static learning and adaptive learning. The first 40% of the 97 277 normal examples were used for training, and the rest for testing. The testing data set also included attacks interspersed into the normal examples with the probability of 1%. The model learned from the training examples was applied to the testing data set. The model remained unchanged during the testing process for static learning, while it was updated continuously for adaptive learning methods. Table 1 lists the numbers of normal and attack examples in Exp. 1 and Exp. 2.

Exp. 1		Exp. 2		
Normal	Attacks	Training normal	Testing normal	Attacks
97 277	998	38 910	58 367	580

Numbers of normal and attack examples in Exp. 1 and Exp. 2

Table 2

The performance (false positive rate, hit rate and cost) of Fuzzy ART and EFuNN with the Exp. 1 data stream

ρ	Fuzzy ART			EFuNN		
	False positive rate (%)	Hit rate (%)	Cost	False positive rate (%)	Hit rate (%)	Cost
0.90	1.82	79.8	0.311	0.259	33.4	0.682
0.91	2.07	73.6	0.389	0.340	37.2	0.649
0.92	2.06	66.3	0.460	0.421	39.3	0.632
0.93	2.35	86.3	0.278	0.573	66.4	0.370
0.94	2.31	66.9	0.469	0.823	57.4	0.475
0.95	3.13	66.6	0.521	1.29	74.9	0.328
0.96	3.33	64.4	0.556	1.97	90.0	0.218
0.97	4.42	89.7	0.369	3.30	91.7	0.281
0.98	5.81	93.2	0.417	6.99	98.7	0.433
0.99	8.84	98.1	0.549	18.3	99.6	1.10

Results illustrate the impact of varying  $\rho$  on their performance.

# 4.3.1. Effectiveness of varying vigilance

The *vigilance* parameter  $\rho$  controls the degree of mismatch between new instances and previously learned patterns. The greater the value of *vigilance*, the more similar the instances ought to be in order to be assigned to a pattern. We studied the effect of varying  $\rho$  while keeping the values of other parameters fixed. Table 2 presents the results when  $\rho$ 's value was varied from 0.9 to 0.99 with the data stream of Exp. 1. The *learning rate* parameter  $\beta$  was set to 0.1,  $N_{watch}$  was 8 and  $Min_{count}$  was 4. The false positive rate was calculated as the percentage of normal instances that were labeled *anomalous* out of the 97 277 normal examples. Similarly, the hit rate was the percentage of detected attacks (i.e., labeled *anomalous*) out of the 998 attacks.

The results show that the false positive rate increases monotonically as the vigilance threshold is raised. This is due to the fact that more normal instances are classified as *uncertain* and then *anomalous* when the value of  $\rho$  increases. Meanwhile, it is interesting to note that the hit rate oscillates at lower  $\rho$  values, and then approaches to 100% as  $\rho$  is raised nearer to 1.0. Ideally, the hit rate should increase

Table 1

monotonically as well. Its oscillation may suggest the abnormality of the data. The cost of Fuzzy ART reaches the lowest value at  $\rho = 0.93$  with a false positive rate of 2.35% and hit rate of 86.3%. For EFuNN, the lowest cost is obtained at  $\rho = 0.96$  while the hit rate is 90% and the false positive rate is as low as 1.97% (values in bold).

# 4.3.2. Effectiveness of varying learning rate

The *learning rate* parameter  $\beta$  determines how fast the system should adapt to new instances in order to accommodate them. A higher value of  $\beta$  places more weight to the new instance when it is assigned to a pattern and less weight to existing members of the pattern. We evaluated the performance of Fuzzy ART and EFuNN with the Exp. 1 data stream by widely varying the *learning rate*. The results are described in Table 3. The *vigilance* parameter was set to 0.93 for Fuzzy ART and 0.96 for EFuNN, respectively, since they provided the lowest cost when the effectiveness of varying vigilance was studied.  $N_{watch}$  was set to 8 and  $Min_{count}$  was 4.

It is interesting to note that for the Exp. 1 data set,  $\beta = 0.1$  appears to be the best choice for both Fuzzy ART and EFuNN in terms of the cost (values in bold). Higher  $\beta$  values provide relatively stable false positive rates and hit rates. For Fuzzy ART, lower  $\beta$  values ( $\beta = 0.01$  or 0.001) cause much lower false positive rates as well as lower hit rates. For EFuNN, however, the false positive rate gets even higher at lower  $\beta$  values while the hit rate declines slightly.

# 4.3.3. Effectiveness of varying N<sub>watch</sub> and Min<sub>count</sub>

Table 3

 $N_{watch}$  and  $Min_{count}$  are two other important parameters for an adaptive anomaly detection system.  $N_{watch}$  represents the delay the system will experience before it evaluates a newly created *uncertain* pattern. If it is too long, there is a risk that an anomalous instance cannot be handled in a timely manner. If it is too short, large amount of false alarms may be generated.  $Min_{count}$  is the minimum number of

β	Fuzzy ART			EFuNN		
	False positive rate (%)	Hit rate (%)	Cost	False positive rate (%)	Hit rate (%)	Cost
0.001	0.256	24.9	0.766	2.34	76.4	0.377
0.01	0.675	54.7	0.493	2.20	88.3	0.249
0.1	2.35	86.3	0.278	1.97	90.0	0.218
0.3	3.17	68.6	0.504	1.69	77.3	0.329
0.5	3.44	71.0	0.496	1.64	72.7	0.371
0.7	3.58	70.1	0.513	1.60	77.4	0.322
0.9	3.53	79.0	0.369	1.47	76.1	0.327
1.0	3.23	67.8	0.515	1.55	74.9	0.343

The performance of Fuzzy ART and EFuNN with the Exp. 1 data stream

Results illustrate the impact of varying  $\beta$  on their performance.

Nwatch	Mincount	Fuzzy ART			EFuNN		
		False positive rate (%)	Hit rate (%)	Cost	False positive rate (%)	Hit rate (%)	Cost
4	2	1.71	74.2	0.360	1.53	88.9	0.203
8	4	2.35	86.3	0.278	1.97	90.0	0.218
12	4	1.77	77.1	0.335	1.65	89.4	0.205
12	6	4.03	70.0	0.542	3.66	92.9	0.291
12	8	6.04	95.4	0.408	5.04	95.7	0.345
16	6	2.97	61.1	0.567	2.95	92.9	0.248
16	8	4.95	72.0	0.577	4.19	94.3	0.309
16	10	6.77	84.2	0.564	6.41	96.3	0.422

The performance of Fuzzy ART	and EFuNN with th	ne Exp. 1 data stream

Results illustrate the impact of varying  $N_{watch}$  and  $Min_{count}$  on their performance.

members that an *uncertain* pattern ought to have before it is changed to *normal*. We empirically studied the effect of varying  $N_{watch}$  and  $Min_{count}$  on the performance of Fuzzy ART and EFuNN. Different values of  $N_{watch}$  and  $Min_{count}$  and the corresponding results are described in Table 4. The *vigilance* parameter was set to 0.93 for Fuzzy ART and 0.96 for EFuNN, and the *learning rate* was 0.1 for both of them.

The results show that  $N_{watch} = 8$  and  $Min_{count} = 4$  is a better choice than others for Fuzzy ART as it provides the lowest cost. Similarly,  $N_{watch} = 4$  and  $Min_{count} = 2$  gives the best performance for EFuNN. The hit rate of EFuNN is higher and more stable than that of Fuzzy ART as the values of  $N_{watch}$  and  $Min_{count}$  change. It indicates that given the distance measure of EFuNN, the attacks are more distinguishable among the normal instances (values in bold).

## 4.3.4. Static learning vs. adaptive learning

We compared Fuzzy ART and EFuNN with SVM using the Exp. 2 data sets. During the training process, Fuzzy ART and EFuNN assumed every pattern was normal and no instance was discarded. During the testing process, however, the task of Fuzzy ART and EFuNN became twofold: evolving their structure to accommodate new patterns and detecting anomalous instances. For simplicity, we set  $N_{watch}$  to 8 and  $Min_{count}$  to 4. We then varied the *vigilance* parameter's value from 0.9 to 0.99, and the *learning rate*'s value from 0.01 to 0.9. The parameter settings that provide the lowest cost for Fuzzy ART and EFuNN are shown in Table 5.

The SVM model learned from the one-class training data set was applied to the testing data set. Common types of kernel functions used in SVM include linear, radial basis and polynomial functions. In our experiments, we found the radial basis kernel performed better than other kernel functions for one-class learning. The parameter v (Schölkopf et al., 2001), which controls the number of support vectors and errors, was determined by cross-validation with the training data sets.

Table 4

	SVM	Fuzzy ART	EFuNN
ρ		0.93	0.96
β		0.2	0.01
False positive rate (%)	12.4	2.98	0.884
Hit rate (%)	90.7	94.0	85.0
Cost	0.836	0.239	0.203

Table 5 The performance of SVM, Fuzzy ART and EFuNN in Exp. 2

Table 5 compares the performance of SVM, Fuzzy ART and EFuNN. SVM was able to detect 90% of the attacks in the testing data set. However, the false positive rate was as high as 12.4%, which indicates the presence of concept drift between the training data set and the testing data set. Compared to SVM, Fuzzy ART and EFuNN generated significantly less false alarms. Fuzzy ART was the best in terms of hit rate, whereas EFuNN gave the lowest cost.

# 4.4. Masquerade detection with the user profiling data

#### 4.4.1. Data set descriptions

We obtained a set of Windows NT user profiling data from an NSA officer. The data was collected for 20 users on 21 different hosts in a real-world government agency environment (a single user might have worked on multiple hosts). During the raw data collection, a tool was developed to query the Windows NT process table periodically (2-3 times per second) and collect all the process information of each user's login session. Processes that were not related to user identification were filtered out during the pre-processing. The processes that correspond to the windows the user activated are of special interest to us because they represent the programs the user was running. The accumulated CPU time was calculated for each of these window-associated processes from a user's login to logout, which reflects the workload the user performed during this login session. For processes that have the same process name, their CPU times were added together. Then a CPU time vector can be formed for each login session, where the value of each entry is the percentage of CPU time consumed by a unique process during this login session. There are 105 processes contained in this data set, for example, netscape, explorer, outlook, msoffice and so on, while each individual user has his or her own process "vocabulary."

In addition to the CPU information, we also included the login time in the input vectors. A user's login time was categorized as *early morning* (before 7 AM), *morning* (between 7 AM and 12 PM), *afternoon* (between 12 and 6 PM) or *evening* (later than 6 PM). Since the login time has four possible values, we have four entries in the input vectors corresponding to the login time. Therefore, each login session is encoded as a vector that contains CPU time percentages consumed by the user as well as four additional entries that represent the user's login time.

User ID	Exp. 3		Exp. 4		
	Self-sessions	Masquerades	Training	Testing self-sessions	Masquerades
1	184	7	92	92	7
2	54	1	27	27	2
4	45	2	22	23	2
6	55	2	27	28	3
7	50	2	25	25	2
14	58	2	29	29	3
19	87	6	43	44	6
Total	527	22	259	268	25

Number of login sessions and the masquerade examples

We selected the 7 users that have the most login sessions to serve as our masquerade targets. We then used the remaining 13 users as masqueraders and inserted their data into the data of the 7 users. Two experiments were conducted with the data set, similar to the ones with the KDD Cup 1999 data set. The first experiment (Exp. 3) was designed to test our evolving connectionist systems. The masquerade examples randomly drawn from the 13 users were embedded into the 7 users' login sessions with a 3% probability. In the second experiment (Exp. 4), in order to compare the performance of SVM, Fuzzy ART and EFuNN, the 7 normal users' login sessions were split into two parts. The first half of the login sessions were used for training the learning systems, and the remaining for testing. Then the masquerade examples were inserted in the testing data sets with a probability of 6%. The values of the masquerade probability were chosen so that there was at least one masquerade example in each user's testing data set for both experiments. Table 6 shows the numbers of the 7 users' login sessions and the corresponding masquerade examples for each experiment. The user IDs are user identification numbers inherited from the original data set. We built learning models for each of the 7 individual users to see if they could identify the masquerade examples hiding in their testing data sets.

## 4.4.2. Results

For Fuzzy ART and EFuNN, when a testing instance is labeled *anomalous* and a *Level 2 alarm* is generated, it is either a true positive (i.e., a hit) if the instance represents a masquerade example, or a false positive if the instance is the user's own login session. We varied *vigilance*  $\rho$ 's value from 0.89 to 0.99 and *learning rate*  $\beta$ 's values from 0.1 to 1.0, respectively. The choice of  $N_{watch}$  and  $Min_{count}$  depends on the profiled individual user. For simplicity, we set  $N_{watch}$  to 3 and  $Min_{count}$  to 2 for all users, which were found to be an acceptable compromise. For each of the 7 users, we report the parameter settings ( $\rho$  and  $\beta$ ) that give the lowest cost in Table 7 for Exp. 3 and Table 8 for Exp. 4.

In Exp. 3, both Fuzzy ART and EFuNN were able to model user behavior starting from an empty set of normal patterns and still recognize the majority of the

Table 6

UserID	Fuzzy	ART			EFuNI	N		
	ρ	β	False positives	Hits	ρ	β	False positives	Hits
1	0.95	0.4	5	3	0.94	0.2	7	5
2	0.92	0.8	6	1	0.91	0.8	5	1
4	0.90	0.4	1	1	0.89	0.2	2	2
6	0.91 0.4 1 2		0.90	0.4	0	2		
7	0.90 0.6 4 1		1	0.90	0.2	0	1	
14	0.93	0.8	7	1	0.90	0.4	2	1
19	0.91	1.0	0	6	0.89	1.0	2	6
Total			24	15			18	18
	False r	ositive r	ate = $24/527 = 4.55$	5%	False p	ositive r	ate $= 18/527 = 3.42$	%
Overall	Hit rat	e = 15/2	22 = 68.2%		Hit rat	e = 18/2	22 = 81.8%	
		= 0.591				= 0.387		

Table 7 The performance of Fuzzy ART and EFuNN in Exp. 3.  $N_{watch} = 3$ ,  $Min_{count} = 2$ 

masquerade instances, while the false positive rate was under 5%. EFuNN performed slightly better than Fuzzy ART because EFuNN provided higher hit rate and lower false positive rate and thus lower overall cost.

Table 8 shows the performance comparison of static learning with SVM and adaptive learning with Fuzzy ART and EFuNN in Exp. 4. The parameter v of SVM was again determined by cross validation with the training data sets. SVM generated 42 false alarms (15.7% false positive rate) due to the concept drift between the training data set and the testing data set. The false positive rates of Fuzzy ART and EFuNN were significantly less because they were able to adapt to user behavior changes incrementally, while the hit rates were comparable to that of SVM. Therefore, the overall cost of Fuzzy ART and EFuNN was largely reduced.

#### 5. Discussion

Our approach assumes that the number of normal instances vastly outnumbers the number of anomalies, and the anomalous activities appear as outliers in the data. This approach would miss the attacks or masquerades if the underlying assumptions do not hold. For example, some DoS attacks would not be identified by our adaptive anomaly detection systems. Nevertheless, our anomaly detection framework can be easily extended to incorporate signature detection. Previously learned patterns can be labeled in such a way that certain patterns may generate an alert no matter how frequently they are observed, while other patterns do not trigger an alarm even if they are rarely seen (Valdes, 2003).

With our adaptive anomaly detection framework, it is possible that one can deliberately cover his malicious activities by slowly changing his behavior patterns without triggering a *level 2 alarm*. However, a *level 1 alarm* is issued whenever a new pattern is being formed. It is then the security analyst's responsibility to identify the

UserID	SVM		Fuzzy ART	RT			EFuNN			
	False positives F	Hits	φ	β	False positives	Hits	φ	β	False positives	Hits
-	11	6	0.92	1.0	6	7	0.96	0.1	2	9
7	7	1	0.91	1.0	0	1	0.90	1.0	0	1
4	σ	2	0.91	0.6	0	7	0.91	0.8	0	1
9	σ	2	0.89	0.8	1	7	0.94	0.6	1	ю
7	8	2	0.89	0.6	1	1	0.90	0.6	1	1
14	6	1	0.91	1.0	2	1	0.91	0.4	3	1
19	4	9	0.91	0.4	0	9	0.92	0.2	0	9
Total	42 2	20			10	20			7	19
	False positive rate = $42/268 = 15.7\%$	= 15.7%	False po	sitive rate	False positive rate $= 10/268 = 3.73\%$		False po	sitive rate	False positive rate $= 7/268 = 2.61\%$	
Overall	Hit rate $= 20/25 = 80.0\%$ Cost $= 1.14$		Hit rate $= 20/2$ Cost $= 0.424$	Hit rate $= 20/25 = 80.0\%$ Cost $= 0.424$	= 80.0%		Hit rate = $19/2$ Cost = $0.397$	Hit rate $= 19/25 = 76.0\%$ Cost $= 0.397$	= 76.0%	

Table 8 The performance of SVM, Fuzzy ART and EFuNN in Exp. 4

78

user's intent in order to distinguish malicious from non-malicious anomalies, which is beyond the scope of this paper. We also note that in case a continuing abnormal activity occurs, large amount of *level 1 alarms* may be raised and the security analyst can still get overwhelmed.

# 6. Conclusion

This paper has presented a new adaptive anomaly detection framework through the use of evolving connectionist systems. A subject's normal behavior is learned in the online unsupervised mode. The performance of two adaptive anomaly detection systems, based on Fuzzy ART and EFuNN, was empirically tested with the KDD Cup 1999 network data and the user profiling data. The experiments have shown that our adaptive anomaly detection systems are able to adapt to user or network behavior changes while still recognizing anomalous activities. Compared to the SVM-based static learning, the adaptive anomaly detection methods can significantly reduce the false alarms.

In order to make an adaptive anomaly detection system scalable, it might be necessary to prune or aggregate pattern nodes as the system evolves, which is a significant issue for our future work. Other issues of our future work include exploring automated determination of the parameters and comparing more evolving connectionist systems, such as evolving self-organizing maps.

# Acknowledgments

The authors wish to thank Dr. Tom Goldring of NSA for providing us the Windows NT user profiling data set. This work is supported in part by the AFOSR Grant F49620-01-1-0327 to the Center for Digital Security of the University of California, Davis.

#### References

- Carpenter GA, Grossberg S., editors. Pattern recognition by self-organizing neural networks. Cambridge, MA: MIT Press; 1991.
- Carpenter GA, Grossberg S, Rosen DB. Fuzzy art: fast stable learning and categorization of analog patterns by an adaptive resonance system. Neural Networks 1991;4:759–71.
- Chang C-C, Lin C-J. LIBSVM: a library for support vector machines, 2001. URL http:// www.csie.ntu.edu.tw/cjlin/libsvm
- Debar H, Becker M, Siboni D. A neural network component for an intrusion detection system. In: Proceedings of the 1992 IEEE symposium on security and privacy. Silver Spring, MD: IEEE Computer Society; 1992. p. 240.
- Eskin E. Anomaly detection over noisy data using learned probability distributions. In: Proceedings 17th international conference on machine learning. San Francisco, CA: Morgan Kaufmann; 2000. p. 255–62.
- Eskin E, Arnold A, Prerau M, Portnoy L, Stolfo S. A geometric framework for unsupervised anomaly detection: detecting intrusions in unlabeled data. In: Barbara D, Jajodia S., editors. Applications of data mining in computer security. Dordrecht: Kluwer; 2002.

- Eskin E, Miller M, Zhong Z-D, Yi G, Lee W-A, Stolfo S. Adaptive model generation for intrusion detection systems. In: Proceedings of the ACMCCS workshop on intrusion detection and prevention. Athens, Greece, 2000.
- Fan W. Cost-sensitive, scalable and adaptive learning using ensemble-based methods. PhD thesis, Columbia University, February 2001.
- Forrest S, Hofmeyr SA, Somayaji A, Longstaff TA. A sense of self for unix processes. In: Proceedings of the 1996 IEEE symposium on security and privacy. Silver Spring, MD: IEEE Computer Society; 1996. p. 120.
- Ghosh AK, Schwartzbard A. A study in using neural networks for anomaly and misuse detection. In: Proceedings of the eighth USENIX security symposium, 1999.
- Hossain M, Bridges SM. A framework for an adaptive intrusion detection system with data mining. In: Proceedings of the 13th annual Canadian information technology security symposium. Ottawa, Canada, 2001.
- Jang J-SR, Sun C-T, Mizutani E. Neuro-fuzzy and soft computing: a computational approach to learning and machine intelligence. Englewood Cliffs, NJ: Prentice-Hall; 1997.
- Javitz HS, Valdes A. The nides statistical component description and justification, Technical Report, Computer Science Laboratory, SRI International, Menlo Park, CA, March 1994.
- Kasabov N. Evolving fuzzy neural networks for supervised/unsupervised on-line knowledge-based learning. IEEE Trans Man Mach Cybernet—Part B: Cybernet 2001;31(6):902–18.
- Kasabov N. Evolving connectionist systems: methods and applications in bioinformatics, brain study and intelligence machines. Berlin: Springer; 2002.
- Lane T, Brodley CE. Approaches to online learning and concept drift for user identification in computer security. In: Proceedings of the fourth international conference on knowledge discovery and data mining, 1998. p. 259–63.
- Lane T, Brodley CE. Temporal sequence learning and data reduction for anomaly detection. ACM Trans Inf Systems Security 1999;2(3):295–331.
- Lazarevic A, Ertoz L, Kumar V, Ozgur A, Srivastava J. A comparative study of anomaly detection schemes in network intrusion detection. In: Proceedings of the third SIAM international conference on data mining. San Francisco, CA: 2003. p. 25–36.
- Lee W, Stolfo SJ. A framework for constructing features and models for intrusion detection systems. ACM Trans Inf Systems Security 2000;3(4):227–61.
- Liao Y, Vemuri VR. Use of k-nearest neighbor classifier for intrusion detection. Comput Security 2002;21(5):439–48.
- Mahoney MV, Chan PK. Learning nonstationary models of normal network traffic for detecting novel attacks. In: Proceedings of the eighth ACM SIGKDD international conference on knowledge discovery and data mining. New York: ACM Press; 2002. p. 376–85.
- Maxion RA, Townsend TN. Masquerade detection using truncated command lines. In: Proceedings of international conference on dependable systems & networks. Washington DC: 2002. p. 219–28.
- McHugh J. Intrusion and intrusion detection. Int J Inf Security 2001;1(1):14-35.
- Schölkopf B, Platt JC, Schawe-Taylor J, Smola AJ, Williamson RC. Estimating the support of a highdimensional distribution. Neural Comput 2001;13(7):1443–71.
- Sequeira K, Zaki MJ. Admit: anomaly-based data mining for intrusions. In: Proceedings of SIGKDD, 2002. p. 386–95.
- Teng HS, Chen K, Lu SC. Adaptive real-time anomaly detection using inductively generated sequential patterns. In: Proceedings of IEEE symposium on security and privacy. Oakland, CA: 1990. p. 278–84.
- The third international knowledge discovery and data mining tools competition (kdd cup 1999) data, 1999. URL http://kdd.ics.uci.edu/databases/kddcup99/kddcup99.html
- Valdes A. Detecting novel scans through pattern anomaly detection. In: Proceedings of DARPA information survivability conference and exposition. Washington, DC: 2003. p. 140–51.
- Vapnik V. Statistical learning theory. New York: Wiley; 1998.
- Yamanishi K, Takeuchi J, Williams G. On-line unsupervised outlier detection using finite mixtures with discounting learning algorithms. Data Mining and Knowledge Discovery 2004;8:275–300.